Drivetech, Inc.

Innovations in Motor Control, Drives, and Power Electronics

Dal Y. Ohm, Ph.D. - President
25492 Carrington Drive, South Riding, Virginia 20152
Ph: (703) 327-2797 Fax: (703) 327-2747 ohm@drivetechinc.com www.drivetechinc.com

Feb. 19, 2008

Proper Tuning of PI regulators on Servo Systems

Dal Y. Ohm, Drivetech, Inc.

The objective of this report is to illustrate simple methods of implementing and tuning the servo system controlled by nested PI regulators without complicated system modeling. These methods should offer reasonable performance and satisfactory response in many typical systems. Keep in mind that for a system with demanding performance, or associated with a complex dynamics, a more elaborate modeling and compensator design procedure should be exercised to optimize the performance. As an example, current-loop tuning of Azimuth axis was illustrated at the end.

A. Nested Feedback Structure

Many DC servo systems use nested feedback structure as shown in Fig. 1. For brushless motors, electronic commutation makes the system slightly more complex with commutation blocks and multiple current regulators but the fundamental concept of feedback control is very much similar. There are many reasons that this nested control structure is preferred in industry. Among them are

(1) This structure costs less to implement than multi-variable control structure, since low bandwidth control such as position and velocity are sampled at a lower rate.

(2) PI regulators are simple to tune without complete modeling and are very popular in industry. In addition, it is simple to implement either in analog or digital form.

(3) Retuning when certain parameters are changed is very simple. Two of the frequently changing servo system parameters are motor inductance and inertia, and change of them can be quickly retuned by adjusting proportional gains only in most cases.

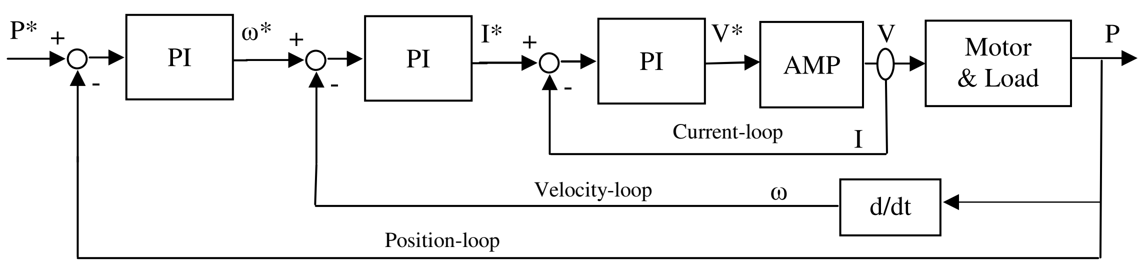


Fig. 1.1 Nested Servo System Structure

B. PID and PI controller

The PI regulator, with 2 tuning parameters works well when the plant behaves like a first-order system. For more complex plants which can be modeled as a 2^{nd} order system, PI control offers only limited performance and the derivative (D) term may be necessary to improve dynamics.

There are several forms when implementing PID regulators [1] such as

(1) Independent $u(s) = (Kp + Ki/s + Kd\ s)\ e(s)$

(2) Standard $u(s) = Kp\ [\ 1 + 1/(Ti\ s) + Td\ s\ ]\ e(s)$

Ti: Integral Time, Td: Derivative Time

(3) Interacting $u(s) = Kp\ [\ 1 + 1/(Ti\ s)][\ 1 + Td\ s\ ]\ e(s)$

Normally, $Ti > 10Td$

In addition, a low-pass filter are often added to the PID (or PI) regulator to reduce noise in the plant or introduced by the derivative term.

In our discussion, we will focus on the standard form PI regulator

$$u(s) = Kp\ [\ 1 + 1/(Ti\ s)]\ e(s), \quad (1)$$

in a slightly different form by introducing a new parameter $\omega i = 1/Ti$ to the above equation as,

$$u(s) = Kp\ (\ s + \omega i)/s)\ e(s). \quad (2)$$

We may call Kp as “proportional gain” and ωi as “integral frequency”. These are two important parameters that we are going to tune on every closed-loop. In this form, ωi specifies a frequency and is independent of loop gain. It will be found out that this standard form is easier to tune and analyze.

C. PI regulator for Current control

Some servo systems run well without current control when desired response time (bandwidth) is not fast such as one Hz or below. When required bandwidth is high, often motor electrical dynamics are interacting with servo system dynamics and the system cannot be tuned with PI or PID control. In this case, we wish to introduce an internal (nested) closed-loop current control so that from the velocity-loop, its response from torque command to torque is ideally fast.

Voltage equation of a typical DC motor, with input voltage v and output current i, is

$$v = (Rs + Ls\ s\ )\ i\ +\ \omega\ \lambda m, \quad (3)$$

where Rs and Ls are armature (or stator in case of Brushless motors) resistance and inductance, respectively. The back emf term, $\omega\ \lambda m$, is considered as an external disturbance and are neglected in the dynamic model. The disturbance may be compensated by adding a feed-forward term but will not be discussed here. As noticed, motor electrical dynamics is a first order system with time constant of $Te = Ls/Rs$. Although the above voltage equation is only for the DC motor, similar electrical equation results, when describing the system in a rotating reference frame, for a brushless motor or vector controlled induction motors. In case of brushless motors, consider v (i) is the magnitude of peak voltage (current) space vector, assuming that commutation is ideal.

In addition, Ls should be interpreted as synchronous inductance (Lq or Ld for buried magnet motors).

For current control, block diagram of the closed-loop system can be modeled as shown in Fig. 2. Note that the plant DC gain K is equal to 1 / Rs.

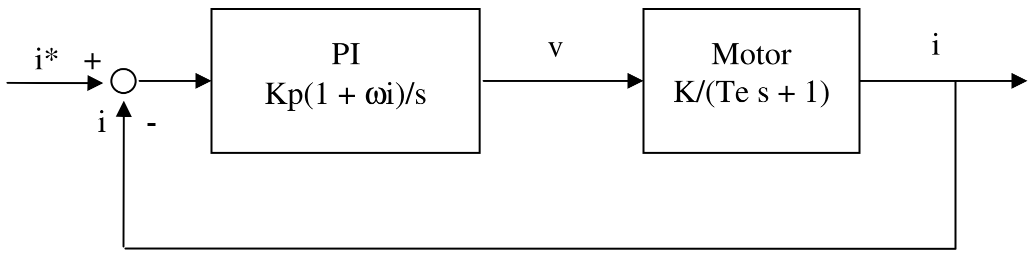


Fig. 2. Closed-loop Model of Current-loop

Now, the closed-loop transfer function can de derived as,

$$Gc(s) = \frac{K\,Kp\,s + K\,Kp\,\omega i}{Te\,s^2 + (1 + K\,Kp)\,s + (K\,Kp\,\omega i)} \qquad (4)$$

The objective is to have the closed-loop bandwidth to be a pre-specified value, ωc. It is a measure of how fast the closed-loop system responds to the changing input command. For many high performance servo systems, ωc is roughly in the range of 500 Hz – 2 kHz.

We have two approaches to tune the system. The first one is called "cancellation tuning" which makes the closed-loop transfer function first order by a pole-zero cancellation. In other words, the closed-loop zero is located at the plant pole location at s = 1/Te. In this case, ωi = 1/Te to cancel the pole. Now, the above transfer function is reduced to a first order system,

$$Gc(s) = \frac{K\,Kp}{Te\,s + K\,Kp} \qquad (5)$$

To make the bandwidth ωc, we should have,

$$Kp = \omega c\,K / Te = \omega c\,Ls \qquad (6)$$

$$\omega i = 1/ Te = Rs / Ls \qquad (7)$$

The second method of tuning is called "pole-placement tuning" which makes two closed-loop poles located at -ωc (similar to critically damped condition in 2nd order all-pole system). In other words, the denominator of Eq. 4 should be

$$Te\,s^2 + (1 + K\,Kp)\,s + (K\,Kp\,\omega i) = Te\,(s^2 + 2\,\omega i\,s + \omega c^2) \qquad (8)$$

Approximate solution of the above equation leads to

$$\omega i = \omega c / 2 \qquad (9)$$

$$Kp = 2\,\omega c\, Ls \qquad (10)$$

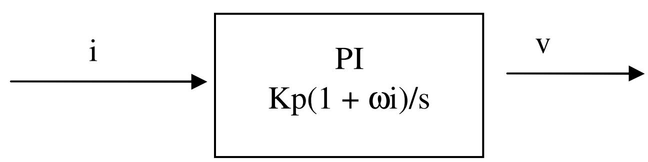
As noticed, proportional gain Kp from cancellation tuning is ½ of that obtained from pole-placement tuning. Considering the fact that closed-loop zeros in Eq. 4 are inherited from the PI regulator zeros and can influence closed-loop dynamics, it makes sense that a more conservative damping by reduction of Kp would be desirable. Practical tuning of both Kp and ωi may fall in between two sets of values.

The above discussion was based on the system described in SI units. In practical systems, input and output variables are scaled differently and may be implemented with digital control. When both input and output units are scaled differently, we can use the following two block diagrams to convert theoretical PI gains into the practical values. In the block diagram, i_max (v_max) is full scale current error (voltage output) in SI unit, while I_max (V_max) is in scaled units of implementation.

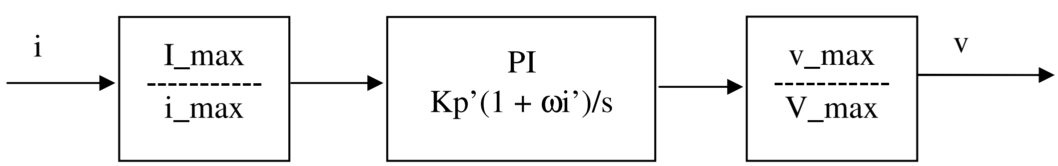
By comparing two systems in Fig. 3, we can derive that

$$Kp' = Kp \frac{i\_max * V\_max}{I\_max * v\_max} \qquad (11)$$

$$\omega i' = \omega i \qquad (12)$$



(A) System with SI units



(B) System with Practical Units

Fig. 3. Comparison of PI gains at two systems of different units

When PI regulator is realized in digital control, a popular regulator structure is in the form of

$$V(n) = Kp' \; \{ Ie(n) + \Sigma\, \omega i'\, Ts\, Ie(k) \} \qquad (13)$$

Where summation is taken from initial time (k=0) to (n-1)th samples of Ie. Here, proportional gain Kp' is identical to its analog equivalent, while integral gain of the above form should be

ωi'Ts. Often, integral term (Kp' Σ ωi' Ts Ie(k) is limited to 50 – 100% of the full scale output value to eliminate controller saturation. Elimination of windup problem may be handled by conditionally integrating (stop integration when V(n) is saturated). An alternative form is,

$$u(n) = u(n-1) + Kp \, [\, e(n) + KiTs \, e(n-1) \,],$$

which does not use integral terms.

One important thing to keep in mind is about the dependency of bandwidth to bus voltage. Although SI unit model is independent of bus voltage as in Fig. 2, when bus voltage is lowered, current-loop bandwidth of the practical system is reduced. This is due to the changes of v_max. To maintain the same bandwidth with fixed Kp, while v_max is decreased, we have to increase Kp'. If we do not increase Kp', then Kp is decreased effectively and results in reduction of the bandwidth.

Another fact in current control system is effects of the back emf term in Eq. 3. When a motor is running at high speeds, the magnitude of the back emf may be close to the supply voltage. In this case, effective voltage to regulate the system is very small and actual system bandwidth at this condition may be significantly lower than the expected bandwidth at stand-still. This is one reason that current-loop bandwidth (at zero speed) should be about 10 or more times higher than desired velocity-loop bandwidth. Another reason is to commutate high excitation frequency properly (in AC motors) to produce maximum torque and maintain high efficiency.

A variation of PI(D) controller structure that introduces one more parameter to control a servo system was discussed in [2] for interest readers.

C. PI regulator for Velocity Control

Assuming that the servo system has a closed-loop current control and its bandwidth is very high compared to desired velocity-loop bandwidth ωv. In that case, motor acts as a linear torque amplifier (T = Kt i) and the plant can be modeled as

$$Kt\, i = Jm\, s + Bm + T_L, \qquad (14)$$

where Kt is the torque constant of the motor, Jm is the total inertia of the motor and the load reflected to the motor shaft, while Bm is the viscous friction constant. In this case, the load torque T_L is considered as an external disturbance and are neglected in the dynamic model. This disturbance may be compensated by adding a feed-forward term but will not be discussed here. As noticed, motor mechanical dynamics is a first order system with time constant of Tm = Jm/Bm. For velocity control, control system block diagram can be simplified as Fig. 4.

The closed-loop transfer function of the above system is,

$$Gc(s) = \frac{Kt\, Kp\, s + Kt\, Kp\, \omega i}{Jm\, s^2 + (Bm + Kt\, Kp)\, s + (Kt\, Kp\, \omega i)} \qquad (15)$$

The objective is to have the closed-loop function bandwidth to be a pre-specified value, ωv. As in current-loop we will take both methods of tuning the system. In the cancellation tuning, we put ωi = 1/Tm. Now, the above transfer function is reduced to a first order system as,

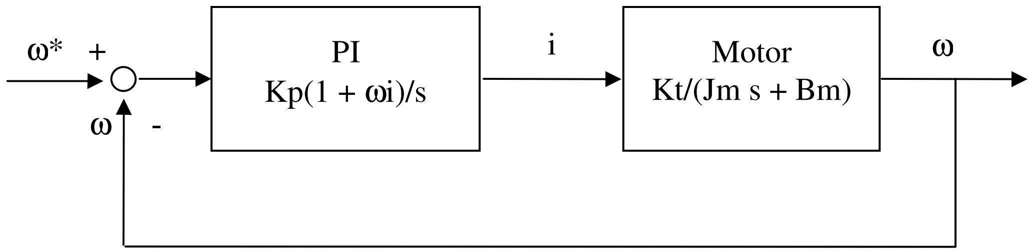


Fig. 4. Closed-loop Model of Voltage-loop

$$Gc(s) = \frac{Kt\ Kp}{Jm\ s + Kt\ Kp} \tag{16}$$

To make the bandwidth ωv, we have

$$Kp = \omega v\ (Jm / Kt) \tag{17}$$

$$\omega i = 1/ Tm = Bm / Jm \tag{18}$$

The strategy of the second method (pole-placement tuning) tries to move two closed-loop poles to $-\omega v$. In other words, the denominator of Eq. 15 should be

$$Jm\ s^2 + (Bm + Kt\ Kp)\ s + (Kt\ Kp\ \omega i) = Jm\ (s^2 + 2\ \omega v\ s + \omega v^2) \tag{19}$$

Approximate solution of the above equation leads to

$$\omega i = \omega v / 2 \tag{20}$$

$$Kp = 2\ \omega c\ (Jm / Kt) \tag{21}$$

As noticed, proportional gain Kp from cancellation tuning is ½ of that obtained from pole-placement tuning. Considering the fact that closed-loop zeros in Eq. 4 are inherited from the PI regulator zeros and can influence closed-loop dynamics, it makes sense that a more conservative damping by reduction of Kp would be desirable. Practical tuning may result in between two set of values.

When both input and output units are scaled differently, instead of SI units, we can convert theoretical PI gains into the practical values. Assuming that ω_max (i_max) is full scale velocity error (current output) in SI unit, while Ω_max (I_max) is corresponding value in scaled units of implementation,

$$Kp' = Kp\ \frac{\omega\_max * I\_max}{\Omega\_max * i\_max} \tag{22}$$

$$\omega i' = \omega i \tag{23}$$

When PI regulator is realized in digital control, a popular regulator structure is in the form of

$$i(n) = Kp' \{ \omega e(n) + \Sigma\ \omega i'\ Ts\ \omega e(k) \} \tag{24}$$

where summation is taken from initial time (k=0) to (n-1)th samples of ωe. Make note that integral gain of the above digital form should be ωi'Ts.

The above velocity control analysis assumes there is no significant additional dynamics (such as torsional resonance) close to the closed-loop servo bandwidth. Controller design with a torsional resonance in the system is very complex and will require more complicated analysis and design procedure.

In many high performance servo systems, ωv of 30 – 100 Hz may be achieved with a good inner current-loop control and resonance-free structure. Some high performance systems based on linear motors and rigid structure may achieve bandwidth of higher than 100 Hz.

E. PI regulator for Position Control

Unlike the velocity-loop, where inner loop bandwidth is much higher than desired velocity-loop bandwidth, desired position-loop bandwidth ωp is closer to inner velocity-loop bandwidth ωv. So the closed-loop position control system can be modeled as in Fig.6. Since the plant is already 2^{nd} order system, we will analyze the system with the proportional control only.

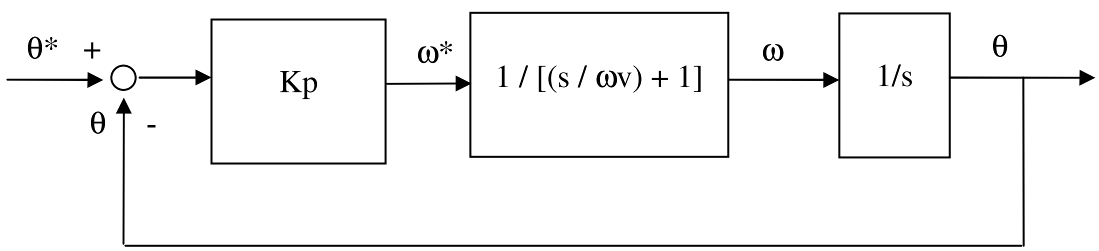


Fig. 6. Closed-loop Model of Position-loop

The, closed-loop transfer function is,

$$Gc(s) = \frac{Kp\ \omega v}{s^2 + \omega v\ s + Kp\ \omega v} \qquad (25)$$

The strategy of the pole-placement tuning moves two closed-loop poles to $-\omega p$. In other words, the denominator of Eq. 15 should be

$$s^2 + \omega v\ s + Kp\ \omega v = s^2 + 2\ \omega p\ s + \omega p^2 \qquad (26)$$

Approximate solution of the above equation leads to

$$\omega p = \omega v / 2 \qquad (20)$$

$$Kp = \omega p^2 / \omega v = (1/2)\ \omega p \qquad (21)$$

Once velocity-loop is closed, position-loop proportional gain is a function of the velocity-loop bandwidth and the position-loop bandwidth ωp may reach almost one half of ωv.

$$\omega i = \omega v / 2 \qquad (20)$$

The position-loop already contains at least one internal integrators (1/s) in the model and integral term may not be necessary. If integral control is desired, integral frequency may be slowly increased (such as 1/10th of ωp) until overshooting response is noticeable.

E. Factors limiting closed-loop bandwidth

Discussion above focused on first order dynamics of the plant which we can alter by applying PI(D) control. The upper limit of the closed-loop bandwidth is determined by the un-modeled fast dynamics in the closed-loop system. Un-modeled dynamics in current-loop may include time delays (sampling time, data conversion time, PWM delay, etc.) and filters (anti-aliasing filter, low-pass filters etc.). Among all fast dynamics, the most significant (lowest frequency pole) dynamics is the major limitation. The maximum closed-loop band width is about 1/3 of the dominant pole frequency.

F. Current Regulator Example

Parameters for a servo drive are as follows.

Rs = 0.9250 (Ohm), Ls = 0.0013 (Henry), Te = 0.0014 (Sec.)

Desired current-loop bandwidth is 2 kHz at 16 kHz sampling

Ts = 1/16000 Sec., ωc = 2*pi*2000

According to the tuning methods, gains are

Kp1 = 16.02, $\omega i1$ = 725.49 (for cancellation tuning)

Kp2 = 32.044, $\omega i2$ = 6283 (for pole-placement tuning)

Units used are as follows.

v_max = 24 (V), V_max = 32767

i_max = 12.9 (A), I_max = 32767.

According to Eq. 11-12,

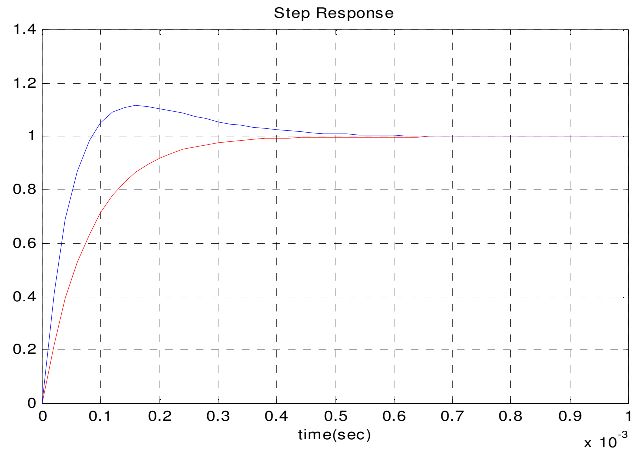
Kp1' = 8.611, Kp2' = 17.22

$\omega i1'$ = 725.49, $\omega i2'$ = 6283

For digital control, integral gain should be

W1idigital = 0.0453, W2idigital = 0.3927

The above tuning was modeled by Matlab and closed-loop poles were calculated. Poles in Hz units are -2000 and -115 Hz (cancellation tuning), and -2542 and -1573 (pole placement). Errors in pole-placement calculation mainly come from approximate calculation of gains. A step responses for the closed-loop system are plotted in the following figure. Response with overshoot is the pole-placement gains (red), while cancellation gains result in no overshoot as expected.



References

[1] K. Astrom and T. Haggulund, "PID Controllers: Theory, Design and Tuning," 2nd Ed., ISA, 1995.

[2] D.Y.Ohm, "Analysis of PID and PDF Compensators for Motion Control Systems," IEEE Industry Applications Society (IAS) Annual Meeting, pp. 1923-1929, Denver, October 2-7, 1994.