PID and PDFF Compensators for Motion Control

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ABSTRACT: *Traditionally in servo systems, the PI(D) controller has been widely used, while other types such as the PDF controller has been applied in some applications. In this article, the PDFF controller which has feedforward term in addition to a conventional PDF(Pseudo-Derivative Feedback) controller is introduced. Tracking and regulation characteristics of the PDFF controller in motion control applications are analyzed in terms of classical frequency domain approach. The PDFF controller is shown to be a generalized PI(D) controller, and relative performance of the well-known cascade PI(D) controller and the PDF controller are dealt with as two special cases. A tuning method of this controller to meet desired performance criteria such as dynamic reference tracking, steady-state error and load torque disturbance rejection capability is discussed along with experimental results.*

I. INTRODUCTION

In high performance motion control applications, a classical multi-loop structure (current loop inside, velocity loop in the middle, and position loop outside) has been widely used, and each loop is controlled by a PID (proportional, integral, and derivative), PI or simply P controller for adequate response. Traditionally, control of the above three control variables has been analyzed and designed separately. For example, the current controller is designed by a power stage design engineer and the velocity control is the responsibility of a drive control engineer, while the position control is designed by a servo engineer at the system integration level. Although this tradition is changing with the advent of intelligent positioning drives using high throughput microprocessors, the multi-loop structure is still favored because of simplicity in control and the natural hierachy of bandwidth requirements. The PI(D) controller has been very popular in industry due to its simple structure and relatively easy tuning procedures with acceptable performance [1]. Several tuning techniques such as Ziegler-Nichols method [2] provide a good intuitive concept to tune the system without knowing the accurate model of a plant. Often, these tuning methods provide satisfactory responses. Although not as popular as PID, the PDF controller has been proposed and used in some applications [3-4]. This is a state-feedback type compensator and has demonstrated a good regulation performance. Before analyzing characteristics of the above controllers, let's consider serveral unique situations encountered in industrial motion control applications including machine tools, factory automation and robotics.

(1) In many motion control applications, plant parameters such as load inertia and friction are not exactly known until the system is installed. In some cases, parameters may vary during operation as the payload changes. Relative variations of these parameters can be quite large (for instance, inertia variation of more than 10 to 1 is not unusual). For this reason, controller parameter tuning must be done on site by users instead of the factory setup. A PI(D) controller or one of its derivatives is a very good choice because of industry-wide familiarity and ease of tuning.

(2) There are several performance requirements for satisfactory motion system response. Two of the most important performance criteria are the reference tracking (or simply tracking) capability in both dynamic and steady-state conditions and the load torque disturbance rejection (regulation) capability. Optimal setup for one requirement may not give satisfactory performance for the other requirement. For motion systems, parameters are usually tuned so that command tracking performances such as rise time, overshoot, and settling time are satisfactory. This tuning may lead to an unsatisfactory response to a step load change (like gravity effect, impact of cutting force, etc.). For a system designed and tuned for a very high degree of load regulation, its tracking performance may not be fast enough.

(3) Often in motion control systems, the system bandwidth is limited by the presence of a torsional resonance of the mechanical system. A compensator that adds zeros to the system may unexpectedly amplify the the resonance unless an additional filtering is introduced.

(4) Most practical motion control systems have limited linear operating ranges. As a result, current and torque outputs saturate whenever the magnitude of the input signal exceeds a certain limit. For meaningful analysis of motion controllers, the amount of control effort must be taken into consideration.

(5) Velocity control performance is more important when an AC motor is used than using with a DC motor. In the DC motor, proportionality relation between armature current and generated torque is reasonably good and maintained during oepration, while in AC motors, the torque and the equivalent armature current are not exactly proportional due to higher torque ripple and coupled dynamics with magnetic flux in the motor. Nonlinearities in torque response and torque transients in the AC motor requires additional effort to the velocity control loop.

(6) Autotuning is gaining popularity in motion control to help users in tuning the parameters. Configuration of controllers must be accompanied by a good autotuning capability.
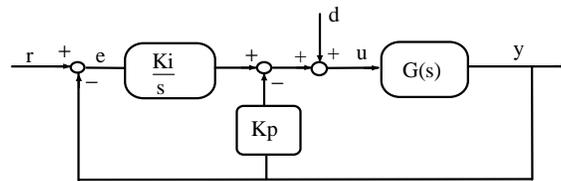
In view of the above situations, the well-known PI(D) or more precisely the "Cascade PI(D)" controller is very attractive in terms of simplicity and popularity. Fig.1.1(A) shows a frequency domain block diagram of a plant with a PI controller of the form

$$u(s) = (Kp + Ki/s) e(s), \qquad\qquad (1.1)$$
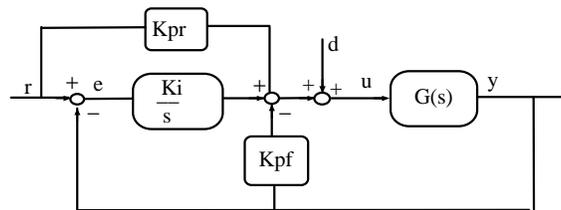
where $e(s) = r(s) - y(s)$ is the error signal, $u(s)$ is the control input to the actuator.
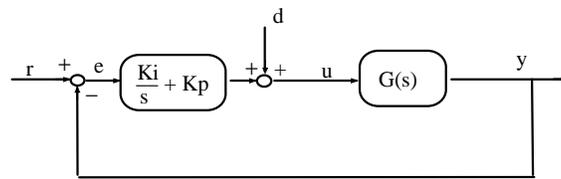
(A) Plant with PI controller

(B) Plant with PDF Controller

(C) Plant with PDFF Controller

Fig.1.1 Various Controller Configurations

It is known that the PI controller does not offer a good disturbance rejection performance compared to other types of controllers. An alternative scheme called the PDF (Pseudo Derivative Feedback) controller [3,4] which can be expressed as

$$u(s) = (Ki/s) e(s) - Kp \, y(s) \qquad\qquad (1.2)$$

has been known for many years. In this scheme, proportional gain Kp is acting only on the output $y(s)$. As in Fig.1.1(B), the PDF controller does not use error signal directly. Another control scheme called the "PDFF controller" (Pseudo-Derivative Control with Feedforward Gain) shown in Fig. 1.1C will be analyzed [3,5] and compared with the above controllers. It has the form of

$$u(s) = (Ki/s) e(s) + Kpr \, r(s) - Kpf \, y(s), \qquad\qquad (1.3)$$

which is the PDF controller with an additional feedforward term. It will be shown that PDFF controller constitues a generalized PI(D) controller, and relative performance of the cascade PI(D) controller and the PDF controller are dealt with as two special (extreme) cases. Throughout the paper, continuous control system approaches are taken, but the results are equally applicable to corresponding digital control systems.

II. PROBLEM FORMULATION

The linear mathematical model of a servo motor and an amplifier connected to a load has been found in many textbooks [6]. Without closed-loop current control, the plant transfer function from voltage command r(s) to motor speed ω(s) can be modelled by a second order system as,

$$G'(s) = \frac{\omega(s)}{r(s)} = \frac{K}{(s + a)(s + c)} \qquad (2.1)$$

where 1/a is called mechanical time constant, while 1/c is called electrical time constant of the plant. When high gain closed-loop current control is implemented as a minor-loop control, two poles are pushed apart, so that mechanical time constant is increased, while electrical time constant is decreased [7]. With high gain current-loop control, the plant can be simplified as a first order model of

$$G(s) = \frac{b}{s + a} \qquad (2.2)$$

where b = K/ c. Here, the coefficient "a" is a relatively small number (slow pole) and may be set to zero (resulting in an integrator) in most inertia dominant applications. For this system, adding a derivative term in a controller has similar effect of readjusting current-loop control, and a derivative feedback is not normally used due to the introduction of noise from obtaining the derivative of the velocity signal, resulting in a simple PI controller as shown in Fig.1.1(A). In this configuration, control action is only based on error signal only. As an alternative control scheme, the PDF controller shown in Fig.1.1(B) is also used in some applications. This is also known as a state feedback, or pole- placement type controller, because this scheme moves the locations of open-loop poles without affecting open-loop zeros. A generalized control structure can be constructed by adding a feedforward term to the PDF controller, as shown in Fig.1.1(C). This configuration will be called the "PDFF" controller following [3]. PDF or PDFF controller may have additional derivative terms, and to distinguish them, the terms "PDF-1 and PDFF-1" will be used for controllers without derivative term, as opposed to "PDF-2 and PDFF-2" for controllers with derivative terms. Discussion on PDFF-2 controllers will be in section IV. In a PDFF-1 controller, if Kpr is set equal to Kpf, the PDFF-1 becomes PI, while Kpr=0 makes the PDF-1 controller. So, the PDFF-1 controller can be regarded as a generalized PI controller. In order to study characteristics of the controllers, a set of transfer functions is derived by selecting various input and output points from Fig.1.1(C) as shown in the following.:

<reference to output>

$$Gc(s) = \frac{y(s)}{r(s)} = \frac{b(Ki + Kpr\, s)}{s^2 + (a + b\, Kpf)s + b\, Ki} \qquad (2.3)$$

<reference to error>

$$Ge(s) = \frac{r(s)-y(s)}{r(s)} = \frac{s^2 + (a + b\, Kpf - b\, Kpr)s}{s^2 + (a + b\, Kpf)s + b\, Ki} \qquad (2.4)$$

<reference to amp input>

$$\frac{u(s)}{} \qquad (Ki + Kpr\, s)\,(s + a)$$

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$$Gu(s) = \frac{}{r(s)} = \frac{}{s^2 + (a + b\,Kpf)s + b\,Ki} \qquad (2.5)$$

<disturbance to output>

$$Gd(s) = \frac{y(s)}{d(s)} = \frac{b\,s}{s^2 + (a + b\,Kpf)s + b\,Ki} \qquad (2.6)$$

For convenience, let's define the ratio

$$p = Kpr / Kpf, \qquad (2.7)$$

for upcoming discussion. We willl consider the range of p between 0 and 1, which is the practical range. Note that all three controllers on Fig.1.1 have identical closed-loop characteristic equations, because the feedforward signal does not affect closed-loop poles. It is apparent that chracteristic differences on the above controllers are solely due to the location of zeros. When Kpf and Ki are tuned so that the characteristic polynomial has two identical roots (critical damping), the closed-loop poles (double) are given by Eq. 2.8, neglecting a small coefficient a.

$$s = -2\,Ki / Kpf. \qquad (2.8)$$

Now, the load disturbance transfer function of Eq. 2.6 is identical for three types of controllers. Load torque disturbance rejection capability is one of the most important specifications in many motion control applications. When a motor is connected to a reduction gear of high ratio, or the load torque disturbance is small enough, torque disturbance may be negligible. But when the motor is connected to the load directly or with low ratio gear, as in direct-drive robot arms or a machine tool spindle drive, and disturbance torque, such as impact of a tool on an unworked piece, is large, then disturbance torque rejection capability is important. For a critically damped second order system, disturbance rejection capability is better if closed-loop system poles are faster (more to the left side in the complex plane). For a system with identical undamped natural frequency (w_n), the effect of disturbance to the output is dependent upon damping ratio. Fig.2.1 shows step disturbance response to the system of Eq.2.6, with fixed Ki, for 5 different values of Kpf (Kpf = 0.5, 0.75, 1.0, 2.0 and 5.0). Plots indicate that the peak magnitude of the response and settling time are dependent on selection of the damping constant. With step disturbance d(s) = Dm/s, Eq.2.6 becomes

$$Y(s) = \frac{b\,Dm}{s^2 + (a + b\,Kpf)s + b\,Ki} \qquad (2.9)$$

Output response can be obtained via Inverse Laplace Transform of the above equation. For overdamped case, where s_1 and s_2 ($s_1 > s_2$) are absolute values of two negative real roots,

$$y(t) = \frac{b\,Dm}{s_1 - s_2} \{ \exp(-s_1 t) - \exp(-s_2 t) \} \qquad (2.10)$$

and for underdamped cases, where m is the real part, and n is the imaginary part of the complex conjugate roots,

$$y(t) = \frac{b\,Dm}{n} \exp(-m t) \sin(n t) \qquad (2.11)$$

Fig. 2.1 shows that peak magnitude of the disturbance response can be reduced by increasing the damping constant. But as damping is increased, a reduced peak magnitude can only be achieved with the expense of slow settling time due to the effect of a slower real pole. A compromise must be made between optimizing peak magnitude and settling time. A normal choice of damping ratio would be within 0.707 - 1.5.
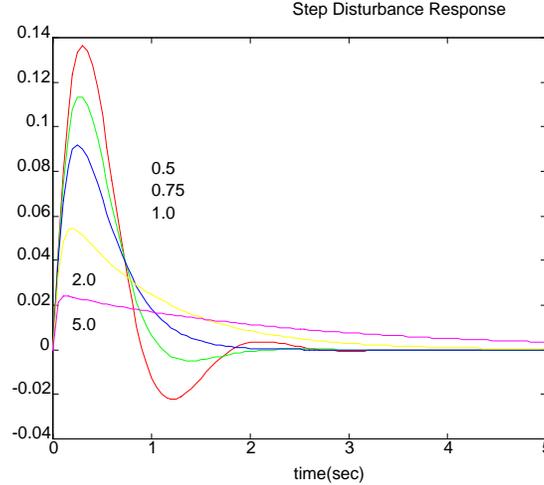
Fig. 2.1 Effect of Damping Factor in Step Disturbance Response

(System of Eq. 2.6 with b=1, Ki = 16)

III. PERFORMANCE COMPARISON

First, consider steady-state error. When the reference input is the unit step, steady-state error can be calculated by applying the final value theorem to Eq. 2.4 as

$$Ess\ (step)\ =\ \lim_{s\to0}\ s\frac{Ge(s)}{s}\ =\ 0. \qquad (3.1)$$

As can be seen from the above equation, zero steady-state error will result for all compensators (regardless of the p value). This is the effect of integral term in the controller. Next, consider the steady-state error when the reference input is changing linearly (constant acceleration). Applying the final value theorem again,

$$Ess\ (ramp)=\ \lim_{s\to0}\ s\ \frac{Ge(s)}{s^2}\ =\ \frac{a + b(Kpf - Kpr)}{b\ Ki}. \qquad (3.2)$$

This error will be very small when the cascade PI controller is used (p=1 or Kpr=Kpf), because the constant a is very small in most motion control applications. The PI controller gives the best steady-state ramp error among all PI controller configurations. As p decreases, steady-state ramp error will be increased, and maximum error will be approximately Kpf / Ki when p=0.

Next, consider the reference to output transfer function Gc(s). As in Eq. 2.3, except PDF controller, a zero is introduced in the closed-loop system. It is well known in control theory that the disturbance response can be tuned by moving closed poles and tracking response can be further optimized by adding zeros to the system via feedforward. Let's look at the system in more detail. Since Gc(s) has zero at

$$z\ =\ -Ki\ /\ Kpr, \qquad (3.3)$$

and poles at the location given by Eq. 2.8 (assuming double poles), it is evident that relative location of the zero to the pole varies as p changes. When p varies from 0 to 1, the zero given by Eq. 3.3 moves from negative infinity to halfway between the pole and the origin. Fig 3.1(A) shows output responses of Gc(s) due to unit step reference input with several different values of p between 0 to 1. All of the other values are fixed (a=1, b=1, Kpf=7 Ki=16) allowing two identical negative real poles. Required actuator input for each output response is depicted in Fig.3.1(B). We can tell from Fig.3.1 that as negative closed-loop zero approaches to the origin, it calls for high instantaneous current (energy) change at the early part of the step, resulting in a fast rise time and a high overshoot. As zero moves toward the left side, its effect is decreased and the maximum energy requirement is smaller, and its time is delayed more. It has been shown that addition of a dominant zero to the system adds overshoot to the system

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and decreases the rise time. In fact, the location of zero set by high p value (p ≈ 1) generates too big an overshoot that is not acceptable in motion control. Practically, to overcome the overshoot, most PI controller gains are tuned so that the system is slightly overdamped. This agrees the well-known observation that PI controller offers better command response, while PDF controller gives better stiffness. The better controller would be the PDFF controller which locates the zero at an optimal place that shortens the step response rise time without overshoot. The p value that leads to the optimal response would be $0 < p < 1$.
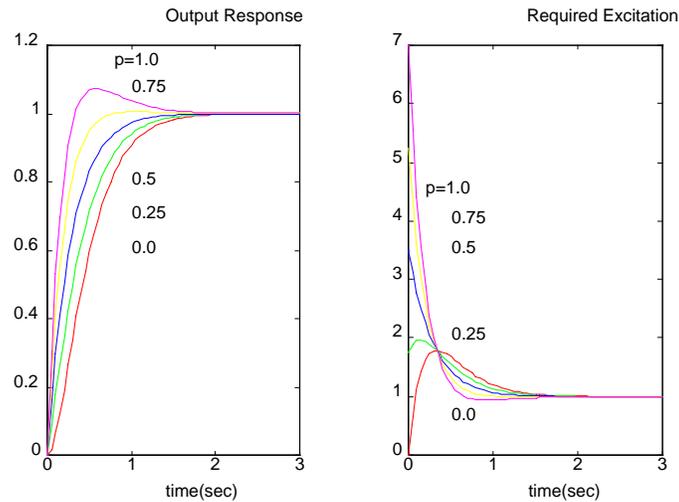

Fig. 3.1 Command Step Response and Required Excitation

(System of Eq. 2.3 with a=1, b=1, Kpf=7, Ki=16)

Since every practical system has a limited energy delivering capability, one test is to see the step responses when a fixed limit of maximum energy call is given for all controllers. In servo systems, operation is within linear range while error is small, but the system saturates when the error becomes larger. The system with larger linear range would be a merit. From Fig. 3.1, we can see that if we limit the peak control input to the same value at 1.75, control gains of PI and PDFF controllers has to be lowered to meet the constraint, and PDF has the fastest response time. This fact indicates that the system with PDF controller has the largest linear operating range [5]. Now this time, let the actuator saturate if the control command is above a set limit. Fig.3.2(A) shows response with the peak torque limit set at the peak torque value of the PDF controller, as indicated by Fig. 3.2(B). This nonlinear characteristic is simulated by applying the 4-th order Runge-Kutta method using MATLAB. Corresponding state-space representation of the system with the PDFF controller is given in Appendix A. In the simulation, all parameters are identical in both controllers except Kpr. In this case, the PI controller gives faster rise time than the PDF even with identical torque limit. This is because of quick consumption of required energy. Except PDF controller, discontinuous control input adds high frequency signal to the system and may excite the torsional resonance. The amount of jerk is highest with PI controller. Although no overshoot is present in this simulation with PI, it is an inherant characteristic in linear region as stated above.

When the cascade PI controller is tuned, "pole-zero cancellation" tuning should be eliminated. When cascade PI controller parameters are tuned, they may be set so that controller zero, -Kpr / Ki is to cancel slow plant pole at -a. This cancellation controller setting is not very desirable because of its poor robustness characteristics. When plant pole is deviated from its nominal value after tuning is done, pole-zero cancellation does not occur, and a slow plant mode appears in the output leading to a sluggish system response. In addition to poor robustness, the cancellation controller leads to a very slow load torque disturbance response, because plant poles are not altered in the disturbance transfer function. A detailed explanation can be found in [8,9]. Also, classical root locus or reference step design rules do not consider disturbance rejection and can sometimes lead to poor regulation.
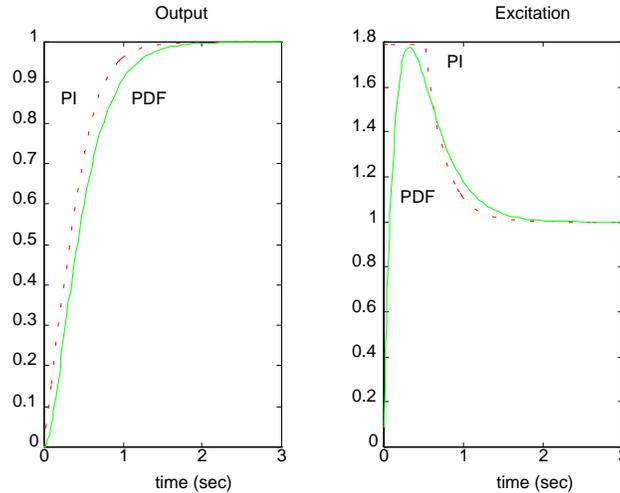
Fig. 3.2 Step Command Responses with Saturation

The above statements are verified by a Baldor Spindle Drive, connected to a 5-HP motor on a test bed. The drive features a digital PDFF-1 controller for velocity control. A more detailed description can be found in [10]. Fig.3.3 shows three reference step responses of the system with PI, PDF and PDFF controller, which are tuned independently of each other to have comparable rise time and overshoots. Note that actual gain values shown in the figures are appropriately scaled in software, but relative magnitudes can be compared to each other. As expected, undamped natural frequencies ($\omega n = \sqrt{b\, Ki}$) were highest in the PDF controller. Step disturbance responses for the above system parameters were tested by short circuiting the load motor while the source motor is running at a constant speed. Response for PI and PDF systems are shown in Fig.3.4, showing very small disturbance magnitude on the output with PDF controller. Disturbance response with PDFF-1 control will be between the two responses.

From the previous analysis, characteristics of PDFF-1 controller can be summarized as follows:

(1) The PI controller (PDFF with p=1) gives the smallest steady-state error during speed change. It offers a very good reference tracking capability compared to PDF.

(2) With no additional zeros imposed, PDF (p = 0) controller generates smooth, continuous control input. In terms of exciting high frequency signals, which may include mechanical resonance of the system, the PDF controller is the best. For disturbance tracking performance, PDF controller is better than PI due to the fact that it does not have to be tuned with overdamping to reduce overshoot as in PI.

(3) For most practical systems, both reference tracking and disturbance rejection capability are important, and the PDFF controller gives flexibility in weighing relative importance of these two performance requirements. The only disadvantage of PDFF controller is that it has 3 parameters to tune. In order to tune PDFF controller, one can start tune the system with p = 0 to tune for disturbance rejection and then tune the feedforward gain (nonzero p) for best reference tracking response without overshoot.

IV. EXTENSION TO HIGHER ORDER MODEL

When velocity control of a motor with voltage amplifier (no current feedback) is desired, or when controlling position without inner velocity-loop control, the plant can be modelled as a second order system. For a second order system, it is known that it is extremely difficult to achieve fast response with a simple PI type controller and often derivative action is added to damp the system oscillation to get a fast response. The analysis described in the previous section can be extended to a second order model by applying PDFF controllers in cascade for each state. For example, a position controlled motion system can be controlled by adding another PDFF position controller to the system whose speed is controlled by the PDFF controller as inb Fig. 1.1C. Since adjusing 6 parameters for a second order plant is too complex and addition of two integraters in the system is not desirable to the system stability, a PDFF-2 controller can be constructed by eliminating one inner integration. A block diagram of a plant with the PDFF-2 controller are depicted in Fig.4.1. Whenever a derivative signal is acquired, high

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frequency noise amplification must to be considered, and an appropriate low pass filtering is necessary in most cases. Since this filter frequency is usually higher than the frequency of our concern, it is simply neglected in our analysis.
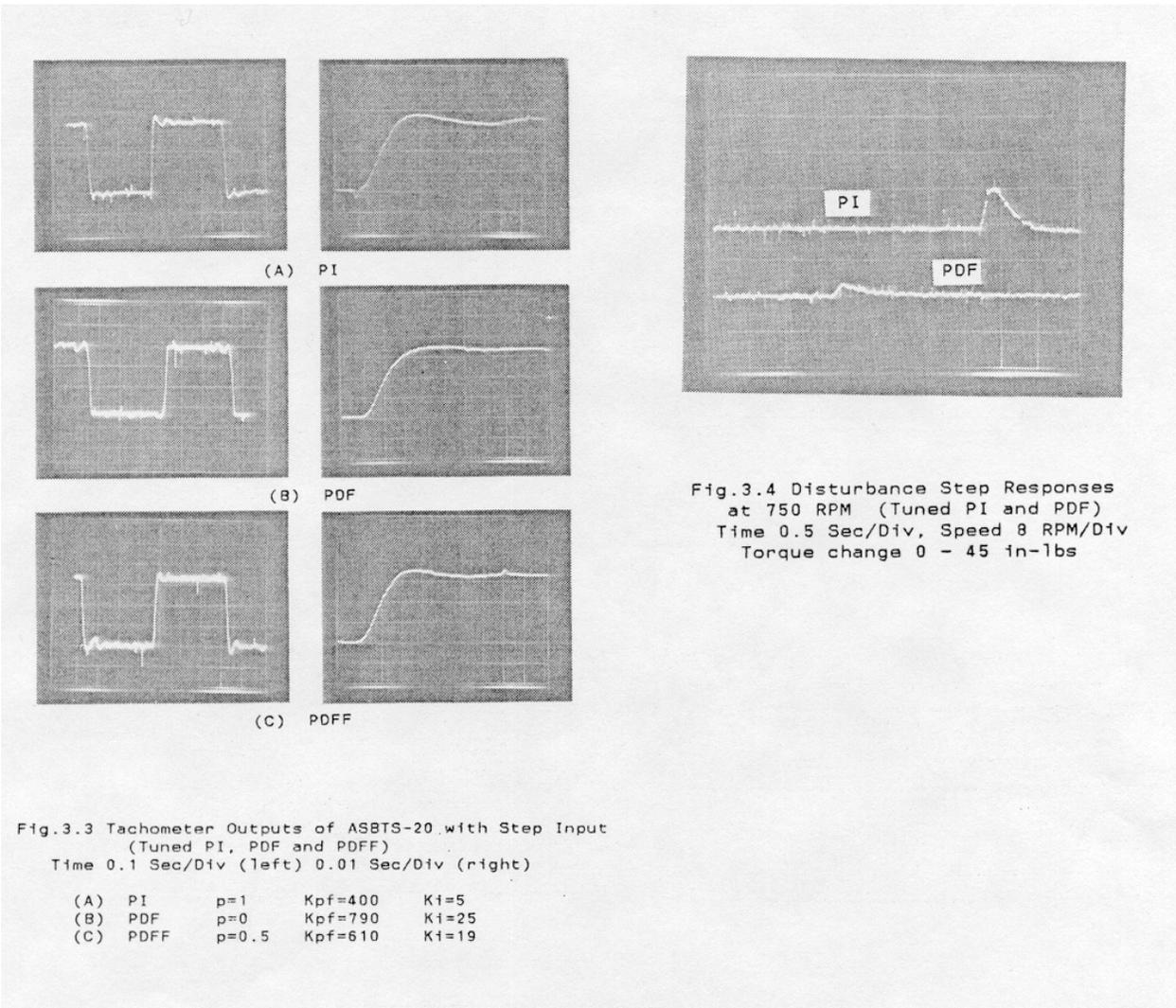
For the PDFF-2 controller, we can define a constant q as the ratio of Kdr to Kdf, corresponding to the definition of constant p. Closed-loop transfer functions correspondent to Eq.2.3 - Eq.2.6 applied to a double integrator system $G(s) = b / s^2$ are as follows.

<reference to output>

$$Gc(s) = \frac{y(s)}{r(s)} = \frac{b\,(Kdr\,s^2 + Kpr\,s + Ki)}{s^3 + b\,Kdf\,s^2 + b\,Kpf\,s + b\,Ki} \qquad (4.1)$$

<reference to error>

$$Gc(s) = \frac{y(s)}{r(s)} = \frac{s\,(s^2 + b\,(Kdf\text{-}Kdr)\,s + b\,(Kpf\text{-}Kpr))}{s^3 + b\,Kdf\,s^2 + b\,Kpf\,s + b\,Ki} \qquad (4.2)$$

(A) PI

(B) PDF

(C) PDFF

Fig.3.3 Tachometer Outputs of ASBTS-20 with Step Input
(Tuned PI, PDF and PDFF)
Time 0.1 Sec/Div (left) 0.01 Sec/Div (right)

(A)	PI	p=1	Kpf=400	Ki=5
(B)	PDF	p=0	Kpf=790	Ki=25
(C)	PDFF	p=0.5	Kpf=610	Ki=19

Fig.3.4 Disturbance Step Responses
at 750 RPM (Tuned PI and PDF)
Time 0.5 Sec/Div, Speed 8 RPM/Div
Torque change 0 - 45 in-lbs

$$Gc(s) = \frac{y(s)}{r(s)} = \frac{b\,s}{s^3 + b\,Kdf\,s^2 + b\,Kpf\,s + b\,Ki} \tag{4.3}$$

Similar results as in the previous sections can be obtained from the above transfer functions. Disturbance response is irrelavant to the two feedforward gains, tracking capability is best with PID ($p = 1$ and $q = 1$), and two zeros in Eq. 4.1 is mandatory in PID control and may not be at a desiable location. For this PDFF-2 controller, tuning may be done without Kpr and Kdr for disturbance rejection first, and then Kpr and Kdr are selected to obtain best tracking capability. For the PDFF-2 controller, location of zeros in the output transfer finction of Eq. 4.1 can be expressed as

$$z = -(Kpr/2\,Kdr) \pm \sqrt{(Kpr/2\,Kdr)^2 - Ki/Kdr}. \tag{4.4}$$

From the above expression, it can be concluded that the practical range of p and q (or Kpr and Kdr) is not restricted to 0 - 1 as in PDFF-1 controller. In fact, if $p = 0$, the zeros are located on the imaginary axis which is not desirable, and this is the reason that proportional action is included in most position control. Also, q value may be increased to a value greater than unity to locate the zeros at an appropriate location. Although the PDFF-2 controller exhibits superior performance in motion control, practically it is complex for general drive users to tune unless the system has autotuning features. From now on, we will discuss about several simplified controllers.
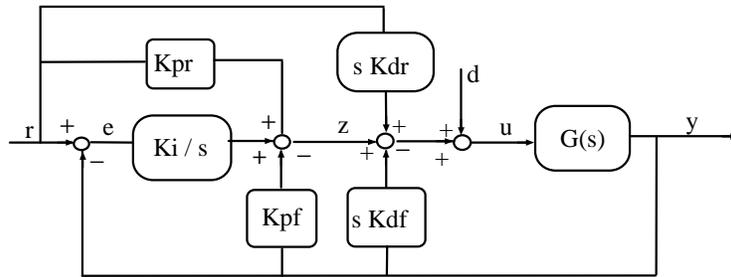

Fig. 4.1 A PDFF-2 Controller

When Kdr=0 and the derivative feedback term Kdf is moved as in Fig.4.2(A), the node z acts as a velocity reference. The addition of an outer PDF control makes P+PDF controller shown in Fig.4.2(A). The cascade PID controller shown in Fig.4.2(B) is another popular controller. Different characteristics between PI and PDF controllers can be extended to the PID and the P+PDF controllers. The P+PDF controller exhibits a very good disturbance rejection capability because only the pole-placement strategy is used, while in the PID controller additional two zeros introduced by the controller may increase overshoot as well as tracking performance. In order to compromise between tracking and regulation requirements, various other configurations are possible. The purpose of the derivative feedback is to move the plant (mechnical) pole to higher frequency (to the left side) so that the plant is closer to a first order model seen from the rest of the controller. For this analysis, assume that the above double integrator plant. With a derivative feedback, we have a modified plant of

$$G_X(s) = \frac{b}{s\,(s + b\,Kd)} \tag{4.5}$$

Clearly, this equation shows that a high derivative gain Kd pushes one pole of the system to a higher frequency and the plant with derivative feedback behaves closer to a first order model. Often this effect can be interpreted as

increased damping. On this modified first order plant, we can now apply any one of PI, PDF or PDFF controller for desired response.
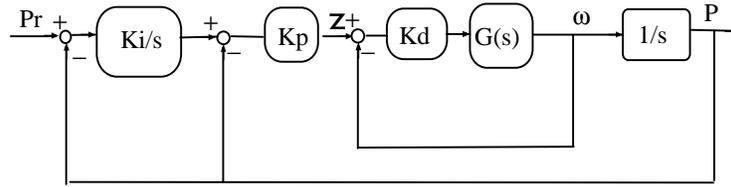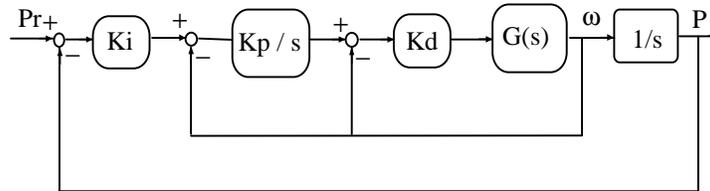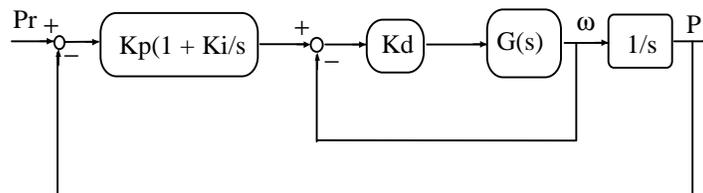

(A) P+ PDF Configuration
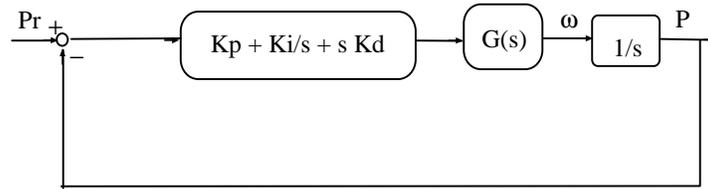
(B) PID   Configuration

Fig.4.2. PDF2 and PID controllers

One other configuration is shown in Fig.4.3(A), called PDF+P controller. It can be easily verified that this controller and P+PDF controller of Fig.3.2(A) give identical transfer function. This configuration for position control is very attractive in a drive offering both position and velocity control, because addition of just one more parameter is required when switching to a position control mode.   Both configurations show a good disturbance rejection characteristics while the tracking performance may not be as good as the PID. Fig.4.3(B) shows P+PI control scheme. In application such as multi-axis position control where the tracking capability is important, this scheme offers a better position tracking performance than the PDF+P configuration.

(A) PDF + P Configuration

(B) P + PI Configuration

Fig.4.3. Other Control Schemes for Second Order Systems.

Now, consider the derivative feedforward Kdr. When Kdr is considered in addition to P+PI configuration of Fig.4.3, Kdr may be used to improve rise time of the velocity performance which enables higher bandwidth of the outer position loop. The effect of Kdr can also be interpreted as relocation of a closed-loop zero to the reference to output transfer function. When PDF outer controller is desired, signals at node z in Fig.4.2(A) will never change abruptly even though reference input is a step. This means that step response of the modified plant may tolerate certain degree of overshoot. So in this case, effect of derivative feedforward action is not as pronounced as in PI outer controller.

V. CONCLUDING REMARKS

Characteristics of a PDFF (a generalized PI(D)) controller for motion control applications are analyzed in terms of location of closed-loop poles and feedforward zeros. By using this scheme, relative performance of the well-known PI and PDF controllers are dealt with as special cases. It has been shown that with an additional flexibility of the PDFF controller, compromising tuning to satisfy two independent performance objectives (tracking and load regulation) is possible. Analysis is extended to a higher order system with the PDFF-2 (with derivative terms) controller and similar results are concluded. Several simplified control structures originated from the PDFF-2 controller are discussed.

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Note: Earlier version of this article has been presented at 1994 IEEE IAS Annual Meeting, pp. 1923-1929.

APPENDIX A. DYNAMIC EQUATION
OF A SATURATED FIRST ORDER SYSTEM WITH
PDFF CONTROLLER

Define the state vector $x(t) = [x1(t) \quad x2(t)]'$ as indicated in the block diagram in Fig. A.1 below. Without considering saturation, the state space representation of the closed-loop system is

$$dx/dt = \begin{bmatrix} -a -b \; Kpf & b \\ -Ki & 0 \end{bmatrix} x + \begin{bmatrix} bKpr \\ Ki \end{bmatrix} r \qquad (A.1)$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

The control input $u(t)$ is subject to a saturation limit so that if the absolute value of $u(t)$ is greater than a set limit, $v(t)$ is limited to the saturated value in magnitude. Practically, this saturation corresponds to the PWM duty cycle of 100 percent in the PWM voltage source amplifier. This nnonlinear dynamic system is simulated with 4-th order Runge-Kutta method.
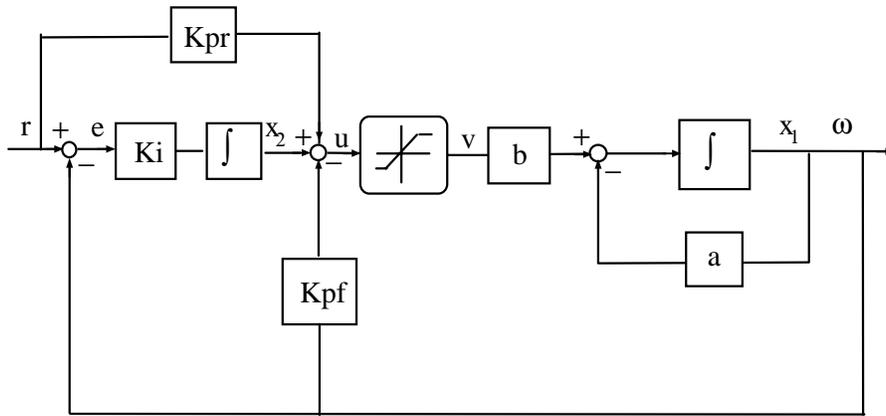

Fig. A.1 Velocity Control System with PDFF controller and Actuator Saturation