DYNAMIC MODEL OF INDUCTION MOTORS
FOR VECTOR CONTROL

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ABSTRACT: *Although traditional per-phase equivalent circuit has been widely used in steady-state analysis and design of induction motors, it is not appropriate to predict dynamic performance of the motor. In order to understand and analyze vector control of induction motors, the dynamic model is necessary. Unfortunately, the dynamic model equations are complex and there are many different forms of the model depending on the choice of reference frame. It is the objective to explain various forms in a concise way to understand clearly. In addition, the fundamental dynamic mechanism of the motor in the synchronous frame is developed and the basic principles of vector control is discussed in general terms.*

I. INTRODUCTION

The induction motor, which is the most widely used motor type in the industry, has been favored because of its good self-starting capability, simple and rugged structure, low cost and reliablilty, etc. Along with variable frequency AC inverters, induction motors are used in many adjustable speed applications which do not require fast dynamic response. The concept of vector control has opened up a new possibility that induction motors can be controlled to achievedynamic performance as good as that of DC or brushless DC motors. In order to understand and analyze vector control, the dynamic model of the induction motor is necessary. It has been found that the dynamic model equations developed on a rotating reference frame is easier to describes the characteristics of induction motors. It is the objective of the article to derive and explain induction motor model in relatively simple terms by using the concept of space vectors and d-q variables. It will be shown that when we choose a synchronous reference frame in which rotor flux lies on the d-axis, dynamic equations of the induction motor is simplified and analogous to a DC motor.

Traditionally in analysis and design of induction motors, the “per-phase equivalent circuit” of induction motors shown in Fig. 1.1 has been widely used. In the circuit, *Rs* (*Rr*) is the stator (rotor) resistance and *Lm* is called the magnetizing inductance of the motor. Note that stator (rotor) inductance *Ls (Lr)* is defined by

$$Ls = Lls + Lm, \quad Lr = Llr + Lm \tag{1.1}$$

where Lls (Lrs) is the stator (rotor) leakage inductance. Also note that in this equivalent circuit, all rotor parameters and variables are not actual quantities but are quantities referred to the stator [1]. Methods of determining circuit parameters from no-load test and locked rotor test are described in [2]. It is also known that induction motors do not rotate synchronously to the excitation frequency. At rated load, the speed of induction motors are slightly (about 2 - 7% slip in many cases) less than the synchronous speed. If the excitation frequency injected into the stator is ω_e and the actual speed converted into electrical frequency unit is ω_o, slip *s* is defined by

$$s = (\omega_e - \omega_o) / \omega_e = \omega_r / \omega_e, \tag{1.2}$$

and ω_r is called the slip frequency which is the frequency of the actual rotor current. In the steady-state AC circuit, current and voltage phasors are used and they are denoted by the underline. In Fig. 1.1, power consumption in the stator is interpreted as $\underline{Is}^2 Rs$, while $\underline{Ir}^2 Rr/s$ represents both power consumption in the rotor and the mechanical ouput (torque). By substracting rotor loss $\underline{Ir}^2 Rr$ from $\underline{Ir}^2 Rr/s$, produced torque (mechanical power divided by the shaft speed) is given by

$$T = \underline{\boldsymbol{Ir}}^2 \, Rr \, (P/2) \; (1\text{-}s) / (s\, \omega_o) = \underline{\boldsymbol{Ir}}^2 \, Rr \, [\, P / (2\, \omega_e)], \tag{1.3}$$

where *P* is the number of poles. Although the per-phase equivalent circuit is useful in analyzing and predicting steady-state performance, it is not applicable to explain dynamic performance of the induction motor. In the next section, we will develop dynamic model of induction motors in general frame work and introduce several equivalent circuits as special cases.

Throughout the article, all vectors are denoted as **boldface** and complex conjugates are denoted by $^{@}$. Vectors on a rotating reference frame is followed by a superscipt letter which designates the frame used as in Vs^{s} (Vs in stationary frame). The derivative operator is denoted by *p* while *P* is the number of poles For notational convenience, let *Y* (scalar) *or* ***Y*** (vector) be the representative notation of any voltage, current or flux linkage variable. Real and Imaginary values of a space vector ***Y*** is denoted by $Re(\boldsymbol{Y})$ and $Im(\boldsymbol{Y})$, respectively. Zero vectors are denoted by ***0*** regardless of the reference frame used.

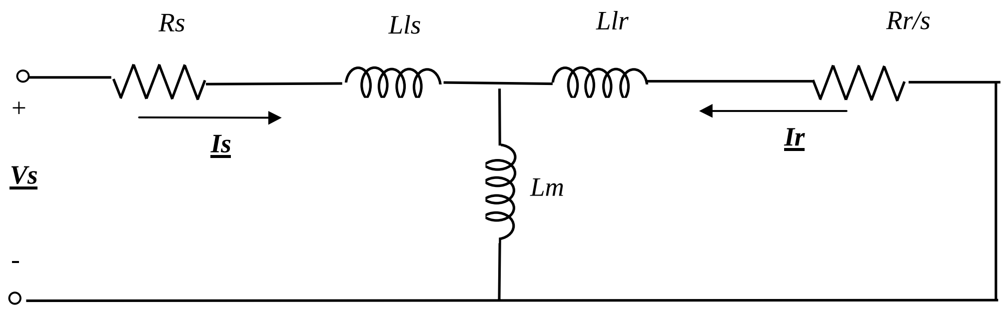


Fig. 1.1 Conventional Per-phase Equivalent Circuit

II. DYNAMIC MODEL IN SPACE VECTOR FORM

In an induction motor, the 3-phase stator windings are designed to produce sinusoidally distributed mmf in space along the airgap periphery. Assuming uniform airgap and neglecting the effects of slot harmonics, distribution of magnetic flux will also be sinusoidal. It is also assumed that the neutral connection of the machine is open so that phase voltages, currents and flux linkages are always balanced and there are no zero phase sequence component in the system. For such machines, the notation in terms of the space vector [3] is very useful. For 3 phase induction motors, the space vector $\boldsymbol{Ys}^{s}$ of the stator voltage, current and flux linkage is defined from its phase quantities by

$$\boldsymbol{Ys}^{s} = (2/3)\,(Ya + \alpha\, Yb + \alpha^{2}\, Yc), \tag{2.1}$$

where $\alpha = exp(j\, 2\pi/3)$. The above transform is reversible and each phase quantities can be calculated from the space vector by,

$$Ya = Re\,(\boldsymbol{Ys}^{s}),\quad Ib = Re\,(\alpha^{2}\, \boldsymbol{Ys}^{s}),\quad Ic = Re\,(\alpha \boldsymbol{Ys}^{s}). \tag{2.2}$$

For a sinusoidal 3-phase quantity of constant rms value, the corresponding space vector is a constant-magnitude vector rotating at the frequency of the sinusoid with respect to the fixed (stationary) reference frame. Note that the space vector is at vector angle 0 when A-phase signal (*Ya*) is at its sinusoidal peak value in steady-state. With space vector notation, voltage equations on the stator and rotor circuits of induction motors are,

$$\boldsymbol{Vs}^{s} = Rs\, \boldsymbol{Is}^{s} + p\, \boldsymbol{\lambda s}^{s} \tag{2.3}$$

$$\boldsymbol{Vr}' = Rr'\, \boldsymbol{Ir}' + p\, \boldsymbol{\lambda r}' = 0 \tag{2.4}$$

It is very convenient to transform actual rotor variables ($\boldsymbol{Vr}'$, $\boldsymbol{Ir}'$, $\boldsymbol{\lambda r}'$) from Eq. 2.4 on a rotor reference frame into a new variables ($\boldsymbol{Vr}^{s}$, $\boldsymbol{Ir}^{s}$, $\boldsymbol{\lambda r}^{s}$) on a stator reference frame as in the derivation of conventional steady-state equivalent circuit. Let the stator to rotor winding turn ratio be *n* and the angular position of the rotor be θ, and define

$$\boldsymbol{Ir}^{s} = (1/n)\, exp(j\,\theta)\, \boldsymbol{Ir}', \quad \boldsymbol{\lambda r}^{s} = n\, exp(j\,\theta)\, \boldsymbol{\lambda r}' \tag{2.5}$$

Also, by defining referred rotor impedances as $Rr = n^{2}\, Rr'$, etc., we have

$$\boldsymbol{Vs}^{s} = Rs\, \boldsymbol{Is}^{s} + p\, \boldsymbol{\lambda s}^{s} \tag{2.6}$$

$$\boldsymbol{0} = Rr\, \boldsymbol{Ir}^{s} + (p - j\omega_{o})\, \boldsymbol{\lambda r}^{s} \tag{2.7}$$

where $\omega_{o} = p\, \theta_{o}$, is the speed of the motor in electrical frequency unit and

$$\boldsymbol{\lambda s}^{s} = Ls\, \boldsymbol{Is}^{s} + Lm\, \boldsymbol{Ir}^{s} \tag{2.8}$$

$$\boldsymbol{\lambda r}^s = Lm\, \boldsymbol{Is}^s + Lr\, \boldsymbol{Ir}^s \tag{2.9}$$

The above 4 equations (Eq. 2.6 - 2.9) constitute a dynamic model of the induction motor on a stationary (stator) reference frame in space vector form. These model equations may be simplified by eliminating flux linkages as

$$\boldsymbol{Vs}^s = (Rs + Ls\, p)\, \boldsymbol{Is}^s + Lm\, p\, \boldsymbol{Ir}^s \tag{2.10}$$

$$\boldsymbol{0} = (Rr + Lr\,(p - j\omega_o))\, \boldsymbol{Ir}^s + Lm\,(p - j\omega_o)\, \boldsymbol{Is}^s. \tag{2.11}$$

From Eqs. 2.10-2.11, The dynamic equivalent circuit model on a stationary reference frame can be drawn as in Fig. 2.1. For steady-state operation with excitation frequency ω_e, p in Eq. 2.10-2.11 may be replaced by $j\omega_e$ and after some algebraic manipulation, we get

$$\boldsymbol{Vs}^s = (Rs + j\omega_e\, Ls)\, \boldsymbol{Is}^s + Lm\, p\, \boldsymbol{Ir}^s \tag{2.12}$$

$$\boldsymbol{0} = (Rr/s + j\omega_e\, Lr)\, \boldsymbol{Ir}^s + j\omega_e\, Lm\, \boldsymbol{Is}^s. \tag{2.13}$$

which exactly describes the conventional steady-state equivalent circuit of Fig. 1.1.

Now, the previous procedure can be generalized so that the dynamic model is described on an arbitrary reference frame rotating at a speed ω_a, where Eq. 2.6 -2.13 is a special case with ω_a,= 0 [4 -5]. To do that, define the new space vector on the arbitrary frame as

$$\boldsymbol{Y}^a = \exp(-j\,\theta_a)\, \boldsymbol{Y}^s \tag{2.14}$$

and reconstruct all the model equations in terms of the new space vectors. In the arbitrary reference frame, Eqs. 2.6-2.7 are modified to

$$\boldsymbol{Vs}^a = (Rs + Ls\, p)\, \boldsymbol{Is}^a + Lm\, p\, \boldsymbol{Ir}^a + j\omega_a\, \boldsymbol{\lambda s}^a \tag{2.15}$$

$$\boldsymbol{0} = (Rr + Lr\, p)\, \boldsymbol{Ir}^a + Lm\, p\, \boldsymbol{Is}^a + j\,(\omega_a - \omega_o)\, \boldsymbol{\lambda r}^a, \tag{2.16}$$

with new flux linkage equations defined by,

$$\boldsymbol{\lambda s}^a = Ls\, \boldsymbol{Is}^a + Lm\, \boldsymbol{Ir}^a \tag{2.17}$$

$$\boldsymbol{\lambda r}^a = Lm\, \boldsymbol{Is}^a + Lr\, \boldsymbol{Ir}^a \tag{2.18}$$

As before, by substituting Eqs. 2.16-2.17 into Eqs. 2.14-2.15, we have

$$\boldsymbol{Vs}^a = (Rs + Ls\,(p + j\omega_a))\, \boldsymbol{Is}^a + Lm\,(p + j\omega_a)\, \boldsymbol{Ir}^a \tag{2.19}$$

$$\boldsymbol{0} = (Rr + Lr\,(p + j\omega_a - j\omega_o))\, \boldsymbol{Ir}^a + Lm\,(p + j\omega_a - j\omega_o)\, \boldsymbol{Is}^a \tag{2.20}$$

where eliminated flux linkage variables are eliminated.

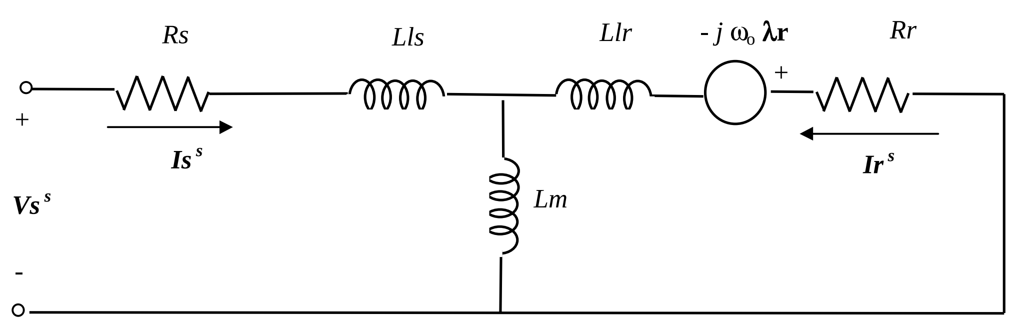


Fig. 2.1 Dynamic Equivalent Circuit on a Stationary Reference Frame

The generalized equivalent circuit on a arbitrarily rotating frame based on Eq. 2.19-2.20 is shown in Fig. 2.2. Now, depending on a specific choice of ω_a, many forms of dynamic equivalent circuit can be established. Among them, the synchronous frame form can be obtained by choosing $\omega_a = \omega_e$. This form is very useful in describing the concept of vector control of induction motors as well as of PM synchronous motors because at this rotating frame, space vector is not rotating, but fixed and have a constant magnitude in steady-state. Since space vectors in the synchronous frame will frequently be used, they are denoted without any superscript indicating the type of frame. Another possible reference frame used in vector control is the rotor reference frame by choosing $\omega_c = \omega_o$ which is , in fact, the reverse step of Eq. 2.5 with n =1.

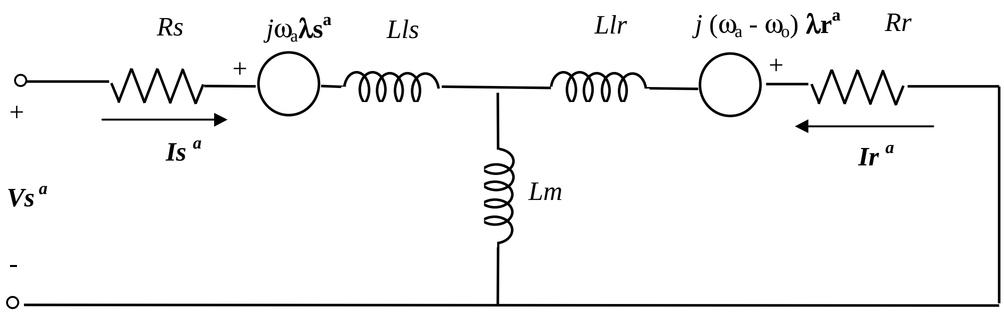


Fig. 2.2 Dynamic Equivalent Circuit on an Arbitray Reference Frame Rotating at ω_a.

III. D-Q EQUIVALENT CIRCUIT

In many cases, analysis of induction motors with space vector model is complicated due to the the fact that we have to deal with variables of complex numbers. For any space vector ***Y***, define two real quantities *Sq* and *Sd* as,

$$\boldsymbol{S} = Sq - j\,Sd \tag{3.1}$$

In other words, $Sq = Re\,(\boldsymbol{S})$ and $Sd = -\,Im\,(\boldsymbol{S})$. Fig. 3.1 illustrates the relationship between d-q axis and complex plane on a rotating frame with respect to stationary a-b-c frame. Note that d- and q-axes are defined on a rotating reference frame at the speed of $\omega_a = p\,\theta_a$ with respect to fixed a-b-c frame.

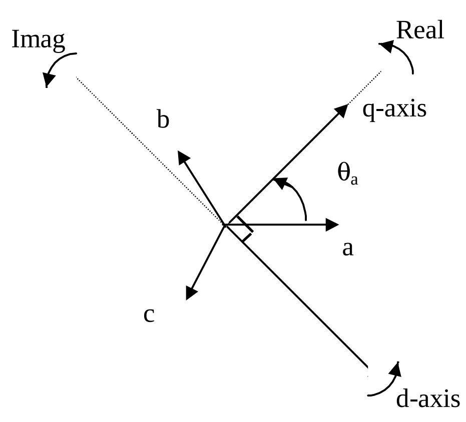


Fig. 3.1 Definition of d-axis and q-axis on an arbitrary reference frame

With the above definition, Eq. 2.19-2.20 can be translated into the following 4 equations of real variables expressed in a matrix form.

$$\begin{bmatrix} Vqs^a \\ Vds^a \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} Rs + Ls\,p & \omega_a\,Ls & Lm\,p & \omega_a\,Lm \\ -\omega_a\,Ls & Rs + Ls\,p & -\omega_a\,Lm & Lm\,p \\ Lm\,p & (\omega_a - \omega_o)\,Lm & Rr + Lr\,p & (\omega_a - \omega_o)\,Lr \\ -(\omega_a - \omega_o)\,Lm & Lm\,p & -(\omega_a - \omega_o)\,Lr & Rr + Lr\,p \end{bmatrix} \begin{bmatrix} Iqs^a \\ Ids^a \\ Iqr^a \\ Idr^a \end{bmatrix} \tag{3.2}$$

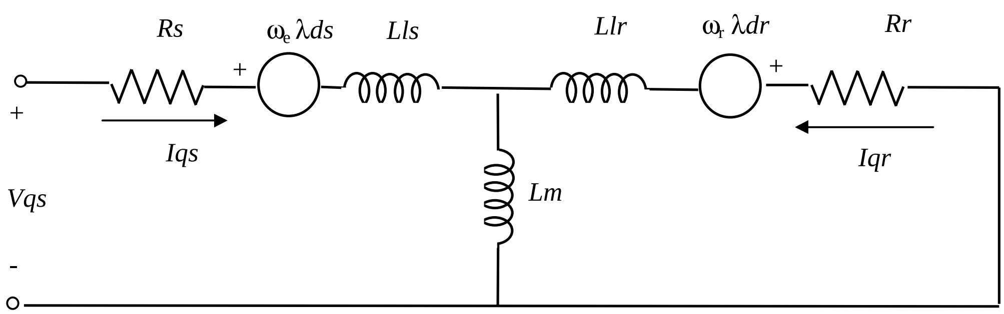
For future reference, the above matrix equation simplified for popular reference frames in analysis and design of vector control will be introduced. For stationary reference frame, by substituting $\omega_a = 0$, the above equation is reduced to

$$\begin{bmatrix} Vqs^s \\ Vds^s \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} Rs + Ls\,p & 0 & Lm\,p & 0 \\ 0 & Rs + Ls\,p & 0 & Lm\,p \\ Lm\,p & -\omega_o Lm & Rr + Lr\,p & -\omega_o Lr \\ \omega_o Lm & Lm\,p & \omega_o Lr & Rr + Lr\,p \end{bmatrix} \begin{bmatrix} Iqs^s \\ Ids^s \\ Iqr^s \\ Idr^s \end{bmatrix} \tag{3.3}$$

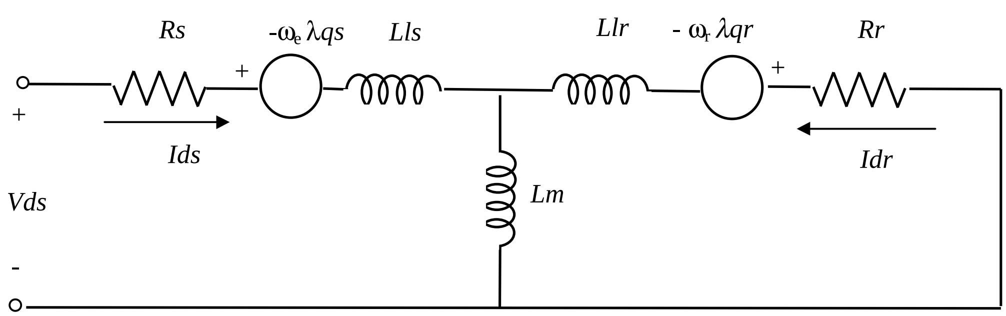
Some implementation of vector drive includes calculation in rotor reference frame (frame is attached to the rotor rotating at ω_o). In this case, we can substitute all ω_a in Eq. 3.2 by ω_o, which makes simplified rotor voltage equations. Moreover, for synchronous frame, we have

$$\begin{bmatrix} Vqs \\ Vds \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} Rs + Ls\,p & \omega_e Ls & Lm\,p & \omega_e Lm \\ -\omega_e Ls & Rs + Ls\,p & -\omega_e Lm & Lm\,p \\ Lm\,p & \omega_r Lm & Rr + Lr\,p & \omega_r Lr \\ -\omega_e Lm & Lm\,p & -\omega_r Lr & Rr + Lr\,p \end{bmatrix} \begin{bmatrix} Iqs \\ Ids \\ Iqr \\ Idr \end{bmatrix} \tag{3.4}$$

As mentioned before, each variable (voltage, current or flux linkage) in the synchronous frame is stationary and fixed to a constant magnitude in steady-state. Based on Eq. 3.4, dynamic d-q equivalent circuit is shown in Fig. 3.2.



(A) Q-axis Circuit



(B) D-axis Circuit

Fig. 3.2 D-Q Equivalent Circuit on a Synchronous Frame

For dynamic simulation of induction motors, Eq. 3.3 or Eq. 3.4 may be used. In this case, one may prefer to use the standard form of differential equation as

$$p\,\mathbf{X} = \mathbf{A}\,\mathbf{X} + \mathbf{B}\,\mathbf{U}. \tag{3.5}$$

For Eq. 3.4, matrix quantities on the above equation are as follows.

$$\mathbf{X} = \begin{bmatrix} Iqs \\ Ids \\ Iqr \\ Idr \end{bmatrix}, \qquad \mathbf{U} = \begin{bmatrix} Vqs \\ Vds \\ 0 \\ 0 \end{bmatrix} \qquad \mathbf{B} = 1/\Delta \begin{bmatrix} \mathrm{Lr} & 0 & -\mathrm{Lm} & 0 \\ 0 & \mathrm{Lr} & 0 & -\mathrm{Lm} \\ -\mathrm{Lm} & 0 & \mathrm{Ls} & 0 \\ 0 & -\mathrm{Lm} & 0 & \mathrm{Ls} \end{bmatrix} \qquad (3.6)$$

and

$$\mathbf{A} = 1/\Delta \begin{bmatrix} Rs\,Lr & \omega_e Ls\,Lr - \omega_r Lm^2 & -\ Rr\,Lm & \omega_o Lm\,Lr \\ \omega_r Lm^2 - \omega_e Ls\,Lr & Rs\,Lr & -\omega_o Lr\,Lm & -Rr\,Lm \\ -Rs\,Lm & -\omega_o Ls\,Lm & Rr\,Ls & \omega_r Ls\,Lr - \omega_e Lm^2 \\ \omega_o Ls\,Lm & -Rs\,Lm & \omega_e Lm^2 - \omega_r Ls\,Lr & RrLs \end{bmatrix} \qquad (3.7)$$

In the above equation, $\Delta = Ls\ Lr - Lm^2$. Although both Eq. 3.4 and Eq. 3.5 are frequently used to describe the induction motor on a synchronous frame, we need another set of equations that include flux linakge variables to explain the concept of vector control. By translating Eq. 2.15 - 2.18 in d-q coordinate on a synchronous frame, we have the following 8 equations. Both stator and rotor voltage equations are,

$$Vqs = Rs\,Iqs + p\,\lambda qs + \omega_s\,\lambda ds \qquad (3.8)$$

$$Vds = Rs\,Ids + p\,\lambda ds - \omega_s\,\lambda qs \qquad (3.9)$$

$$0 = Rr\,Iqr + p\,\lambda qr + \omega_r\,\lambda dr \qquad (3.10)$$

$$0 = Rr\,Idr + p\,\lambda dr - \omega_r\,\lambda qr, \qquad (3.11)$$

where flux linkage variables are defined by

$$\lambda qs = Ls\,Iqs + Lm\,Iqr \qquad (3.12)$$

$$\lambda ds = Ls\,Ids + Lm\,Idr \qquad (3.13)$$

$$\lambda qr = Lm\,Iqs + Lr\,Iqr \qquad (3.14)$$

$$\lambda dr = Lm\,Ids + Lr\,Idr. \qquad (3.15)$$

It will be shown in the next section that the above equation are very useful in explaining the dynamic structure of the motor and the concept of vector control.

When induction motors are controlled by a vector drive, control computation is often done in the synchronous frame. Since actual stator variables either to be generated or to be measured are all in stationary a-b-c frame, frame transform should be executed in the control. The most popular transform is between stationary a-b-c frame quantities to synchronously rotating d-q quantities. Combining Eq. 2.1, Eq. 2.14, and Eq. 3.1, we have

$$Sqs = (2/3)\,Re\{\exp(-j\theta a)\,(Sa + \alpha\,Sb + \alpha^2\,Sc)\} \qquad (3.16)$$

$$Sds = -\,(2/3)\,Im\{\exp(-j\theta a)\,(Sa + \alpha\,Sb + \alpha^2\,Sc)\} \qquad (3.17)$$

Or in a simpler form,

$$\begin{bmatrix} Yq \\ Yd \\ 0 \end{bmatrix} = (2/3) \begin{bmatrix} \cos\theta & \cos(\theta - 2\pi/3) & \cos(\theta + 2\pi/3) \\ \sin\theta & \sin(\theta - 2\pi/3) & \sin(\theta + 2\pi/3) \\ 0.5 & 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} Ya \\ Yb \\ Yc \end{bmatrix} \qquad (3.18)$$

and its inverse transform is given by

$$\begin{bmatrix} Ya \\ Yb \\ Yc \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 1 \\ \cos(\theta - 2\pi/3) & \sin(\theta - 2\pi/3) & 1 \\ \cos(\theta + 2\pi/3) & \sin(\theta + 2\pi/3) & 1 \end{bmatrix} \begin{bmatrix} Yq \\ Yd \\ 0 \end{bmatrix} \qquad (3.19)$$

In vector control drives, Eq. 3.18 is frequently used to convert measured currents and voltages to d-q quantities while Eq. 3.19 may be used to feed command signals to the amplifier. In many modern drives, Eq. 3.19 can be accomplished in a slightly different mechansism such as the space vector modulation [6].

Regardless of reference frame, instantaneous input power can be expressed, in terms of space vectors, by

$$Pi = (3/2)\, Re(\mathbf{Vs}\, \mathbf{Is}^{@}), \tag{3.20}$$

or in terms of d-q variables as

$$Pi = (3/2)\, [\, Vds\, Ids + Vqs\, Iqs\,]. \tag{3.21}$$

The reactive power Qi can also be defined as

$$Qi = (3/2)\, Im(\mathbf{Vs}\, \mathbf{Is}^{@}), \tag{3.22}$$

or in terms of d-q variables as

$$Qi = (3/2)\, [\, Vqs\, Ids - Vds\, Iqs\,]. \tag{3.23}$$

This reactive power can be used in some parameter adaptation methods which automatically corrects the rotor time constant parameter (*Tr*) during steady-state operation.

Now, one simple way of obtaining the output torque is to consider the power associated with speed voltage term on Fig. 2.1 as

$$Po = (3/2)\, \omega_o\, Im\, (\boldsymbol{\lambda}\mathbf{r}\, Ir^{@}). \tag{3.24}$$

Since torque is the above power divided by the rotor speed,

$$To = (3/4)\, P\, Im\, (\boldsymbol{\lambda}\mathbf{r}\, \mathbf{Ir}^{@}), \tag{3.25}$$

where *P* is the number of poles. In terms of *d-q* variables , Eq. 3.25 is

$$To = (3/4)\, P\, \{\lambda qr\, Idr - \lambda dr\, Iqr\} \tag{3.26}$$

Although the torque expression on the above is derived from stationary reference frame, it is true for any other reference frames. Many other forms of torque equations are possible [4]. For example, by substituting flux linkage relation of Eq. 2.9 into Eq. 3.25, we have

$$To = (3/4)\, P\, Lm\, Im(\mathbf{Is}\, \mathbf{Ir}^{@}). \tag{3.27}$$

$$To = (3/4)\, P\, Lm\, \{Iqs\, Idr - Ids\, Iqr\} \tag{3.28}$$

Again, by using Eq. 2.9, we can eliminate ***Ir*** on Eq. 3.27 to get

$$To = (3/4)\, P\, (Lm/Lr)\, Im(\mathbf{Is}\, \boldsymbol{\lambda}\mathbf{r}^{@}). \tag{3.29}$$

$$To = (3/4)\, P\, (Lm/Lr)\, \{Iqs\, \lambda dr - Ids\, \lambda qr\} \tag{3.30}$$

It will be shown later that Eqs. 3.29 - 3.30 are particularly important in vector control because output torque is expressed in terms of stator current and rotor flux linkage.

IV. PRINCIPLES OF VECTOR CONTROL

So far, we have not paid attention to the alignment of the rotating reference frame with respect to the physical coordinate. Noting in Eq. 3.28 that torque is directly proportional to *Iqs* if $\lambda qr = 0$, one can choose the rotating d-axis to be the angle of the rotor flux linkage. In fact, this choice offers a lot of advantages of simplifying control and analysis of the motor. Other choices frequently used in direct vector control are stator flux linkage frame (d-axis is aligned to the stator flux linkage) and airgap flux linkage frame, which will be discussed briefly at the end of the section.

When the motor is driven from an ideal current source amplifier, Eq. 3.8-3.9 are automatically satisfied by the source and can be neglected in the analysis. This is practically true on many PWM voltage amplifiers which have high bandwidth closed-loop current control. Assume that the rotor flux always coincides with the rotating d-axis frame, i.e.,

$$\boldsymbol{\lambda r} = -j\,\lambda dr \tag{4.1}$$

Then we have,

$$\lambda qr = 0, \quad p\,\lambda qr = 0. \tag{4.2}$$

Applying the above conditions to Eq. 3.10-3.11, we have

$$Rr\,Iqr \;+\; \omega_r\,\lambda dr \;=0 \tag{4.3}$$

$$Rr\,Idr \;+\; p\,\lambda dr \;=0 \tag{4.4}$$

Next, substitution of these relations into Eq. 3.14-3.15 yields

$$Iqr \;= -\,(Lm\,/\,Lr)\,Iqs \tag{4.5}$$

$$Idr \;=\; (\lambda dr \;-\; Lm\,Ids\,)\,/\,Lr \tag{4.6}$$

Now, by defining the rotor time constant τ_r, a very, very important constant in induction motor dynamics as,

$$\tau_r \;=\; Lr\,/\,Rr, \tag{4.7}$$

we have the following two equations.

$$\omega_r \;=\; (Lm\,/\,\tau_r\,)\,(Iqs\,/\,\lambda dr\,) \tag{4.8}$$

$$\mathrm{p}\,\lambda\mathrm{dr} \;=\; (1\,/\,\tau_r\,)\,(\,-\,\lambda dr + Lm\,Ids) \tag{4.9}$$

In the mean time, torque expression of Eq. 3.28 is reduced to

$$T \;=\; (3/4)\,P\,(Lm\,/\,Lr)\,\lambda dr\,Iqs \tag{4.10}$$

Based on Eqs. 4.8-4.10, we can draw a block diagram as in Fig. 4.1 of induction motor dynamics when rotor flux field oriented condition (Eq. 4.1) is imposed.

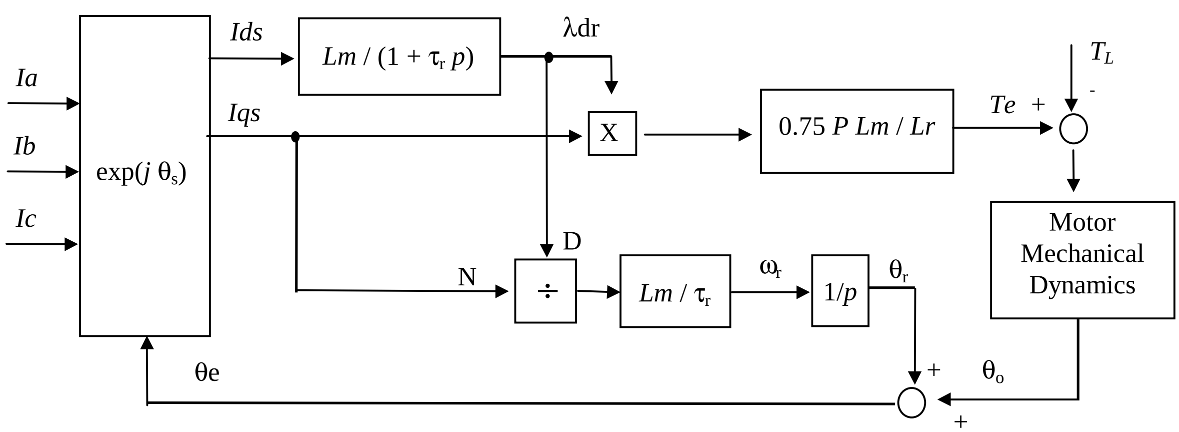


Fig. 4.1 Block Diagram of Induction Motor Dynamics

From the block diagram of Fig. 4.1, we can observe that the output torque is directly proportioal to the q-axis stator current without dynamics while it is subject to a first order dynamics with time constant τ_r from the d-axis stator current. In addition, rotor flux linkage is not affected by the change in *Iqs* (decoupled). We can also see that *Idr* exists only when λdr (Eq. 4.4) changes due to the change in *Ids* (Eq. 4.6). In steady-state, the magnitude of the rotor flux is

$$\lambda dr \;=\; Lm\,Ids \tag{4.11}$$

This situation is analogous to the control characteristics of separately excited DC motors where torque is proportional to armature current, while field flux which has a long time lag due to high inductance of field circuit. Note in the above block diagram that determination of d-axis and q-axis stator currents from 3-phase input currents are based on the rotor flux angle. Inside the motor, rotor flux angle is determined by the angular position of the rotor plus integrated slip frequency which is given by Eq. 4.8.

In vector control of induction motors, the accuracy of rotor flux angle is critical in control because calculation of currents (*Ids, Iqs*) in the synchronous frame is determined by the rotor flux angle. Basically, there are two methods of determing rotor flux angle in vector control. One method, called Indirect Vector Control (IVC) calculates θs from

$$\omega_r^* = (Lm / \tau_r)\,(Iqs^* / \lambda dr^*) \tag{4.12}$$

$$\theta e^* = \theta o + \int \omega_r^*\, dt \tag{4.13}$$

where quantities that are commanded or estimated in drive control are denoted by asterisk (*). Since this method relies on knowlegde of motor parameters such as *Lm* and τ_r, and the real values of which may be changing as operating conditions change, consideration should be given in design to the effects of parameter variations. Another method, called Direct Vector Control (DVC) determines θe* either from the measurement of airgap flux, or from terminal voltages and currents. In the latter case, angle and magnitude of the rotor flux may be calculated by

$$\boldsymbol{\lambda s} = \int (\boldsymbol{Vs} - Rs\, \boldsymbol{Is})\, dt. \tag{4.14}$$

$$\boldsymbol{\lambda r} = (Lm/Lr)\,(\boldsymbol{\lambda s} - Lo\, \boldsymbol{Is}). \tag{4.15}$$

where $Lo = Ls - Lm^2/Lr$. Although DVC may be relatively insensitive to the variations (depending on the actual implementation) of rotor parameters, performance of DVC may be sluggish at low speed operation due to inaccurate knowledge on the stator resistance, integration drift, etc.

In the above discussions, we chose the reference frame d-axis to coincide with rotor flux linkage. This is called “rotor flux orientation.” Sometimes, “stator flux orientation” or “air-gap flux orientation” can be used. The airgap flux space vector is defined by

$$\boldsymbol{\lambda m} = Lm\, \boldsymbol{Is} + Lm\, \boldsymbol{Ir}. \tag{4.16}$$

All three orientation methods use the synchronous reference frame with slight differences in the choice of the reference vector. In any of the above cases, the description up to the previous section for the synchronous frame are applicable. In DVC, stator flux orientation may be used when flux linkage is calculated from terminal voltages and currents, while the airgap flux orientation may be preferred when actual airgap flux sensor is used for direct measurement on the motor. Since we do not have a nice decoupled torque relation shown in Fig.4.1 on both stator and airgap flux orientation methods, additional decoupling compensation should be applied for vector control in either stator or airgap flux orientation. Refer to [7] for further details.

REFERENCES

[1] A. E. Fitzgerald, et al., "Electric Machinery," 5th Ed., McGraw-Hill, 1990.

[2] IEEE Standard 112-1991, "IEEE Standard Test Procedure for Polyphase Induction Motors and Generators," Institute of Electrical and Electronics Engineers, Inc.

[3] G. R. Slemon,"Modelling Induction Machines for Electric Drives," IEEE Trans. on Industry Applications, Vol. 25, No. 6, pp. 1126-1131, Nov. 1989.

[4] D. W. Novotney, et al.(Editor), "Introduction to Field Orientation and High Performance AC Drives," IEEE IAS Tutorial Course, 1986.

[5] A. M. Trzynadlowski, “The Field Orientation Principle in Control of Induction Motors,” Kluwer Academic Publishers, 1994.

[6] J. Holtz, "Pulse Width Modulation for Electronic Power Conversion," Proceedings of IEEE, Vol.82, No.8, pp.1194-1214, Aug. 1994.

[7] R. DeDonker and D. W. Novotney, “The Universal Field Oriented Controller,” IEEE Trans. Industry Applications, Vol. 30, No.1, pp.92-100, Jan. 1994.

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